Torsional Radiation Damping of Arbitrarily Shaped Embedded Foundations

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Abstract: Radiation damping of torsionally oscillating, arbitrarily shaped rigid foundations embedded in linear hysteretic homogeneous soil is studied parametrically with a rigorous boundary element formulation. A simplified analytical model, based on sound physical approximations and calibrated against the numerical results, is also developed. The basemat shapes studied include rectangles of aspect ratios up to 6, circles, triangles, and T-shapes. Particular emphasis is placed on the effects of the depth of embedment and the extent of contact between sidewalls and surrounding soil. It is shown that even foundations placed in an open trench, without sidewall-soil contact, radiate energy more effectively than surface foundations with an identical basemat. Additional radiation damping is generated from vertical sidewalls, even if they are in good contact with the soil only over a small height compared with the embedment depth; hence, separation or slippage between sidewalls and soil near the ground surface would not substantially affect the amount of torsional damping. Physical arguments are put forward to explain these findings and develop insight to the problem. The paper concludes with an illustrative numerical example.

Introduction

This is the second of two papers studying the dynamic response $\theta_z(t) = \theta_z \exp(i\omega t)$ of arbitrarily shaped rigid foundations embedded at depth D in an elastic half-space and subjected to a harmonic torsional excitation $M_z(t) = M_o \exp(i\omega t)$ (Fig. 1). The first paper (Ahmad and Gazetas 1992) studies the static stiffness K_t and the dynamic stiffness coefficient $k_t(\omega)$ of such foundations, and proposes simple algebraic formulas for estimating these quantities. This paper presents information for computing the radiation dashpot coefficient $C_t(\omega)$, which represents the geometric spreading of elastic energy by waves propagating away from the foundation. The foregoing stiffness and damping terms combine as follows to give the dynamic impedance (i.e., the dynamic moment-rotation ratio) of the foundation:

$$\kappa_t = \frac{M_z}{\theta_z} = K_t k_t(\omega) + i\omega C_t(\omega) \quad \text{(1)}$$

The numerical results in this paper, obtained with a rigorous boundary element (BE) algorithm outlined in the first paper (Ahmad and Gazetas 1992), are presented in the form of dimensionless graphs for the dashpot coefficients, covering a wide range of basemat shapes, embedment depths, and frequencies of excitation. The height of contact, d, between foundation sidewalls and surrounding soil is varied parametrically between the two extremes of $d = 0$ (no sidewall-soil contact) and $d = D$ (complete sidewall-

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Note. Discussion open until January 1, 1993. Separate discussions should be submitted for the individual papers in this symposium. To extend the closing date one month, a written request must be filed with the ASCE Manager of Journals. The manuscript for this paper was submitted for review and possible publication on October 2, 1990. This paper is part of the ***Journal of Geotechnical Engineering***, Vol. 118, No. 8, August, 1992. ©ASCE, ISSN 0733-9410/92/0008-1186/$1.00 + $.15 per page. Paper No. 611.

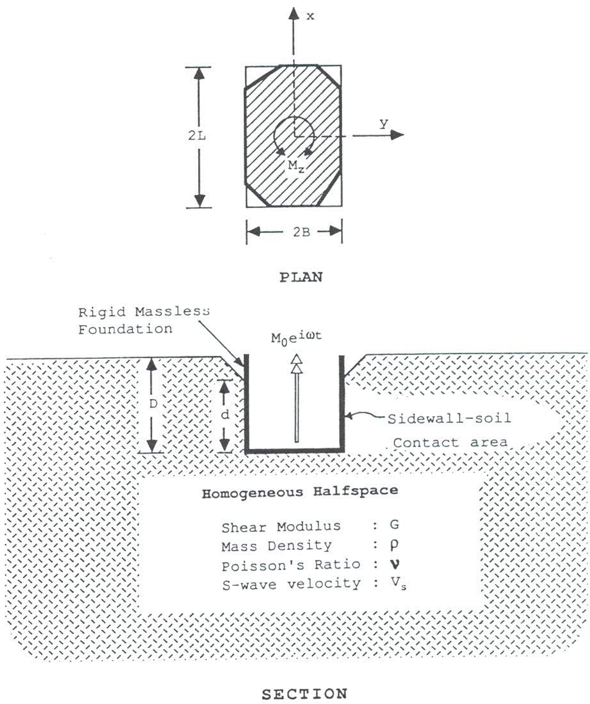


FIG. 1. Problem Geometry and Loading

soil contact over the full depth of embedment). Based on the numerical data, a simple but realistic method involving algebraic formulas is developed for estimating the torsional radiation damping of arbitrarily shaped rigid foundations embedded in a reasonably uniform and deep soil deposit, which can be modeled as a homogeneous, elastic half-space.

In reality, in addition to loss due to radiation, energy is dissipated by hysteretic action in the soil, conveniently expressed through a frequency-independent material damping ratio, β. The combined radiation-hysteretic damping coefficient, $C_t(\omega, \beta)$, can be approximated as

$$C_t(\omega, \beta) \approx C_t(\omega) + \frac{2\beta}{\omega} K_t k_t(\omega) \quad \text{(2)}$$

This approximation is very good for the homogeneous half-space studied herein and quite reasonable for most realistic soil deposits (except, perhaps, at frequencies around the fundamental frequency, $\omega_n = \pi V_s/2H$, of very shallow soil deposits).

Radiation Damping of Surface Foundation

The radiation damping coefficient $C_t(\omega)$ is a measure of the amount of vibration energy transmitted into the supporting soil and carried away by outward and downward propagating waves. A surface foundation subjected to torsion transmits exclusively shear waves into the supporting ground.

These waves originate at every point of the soil-basemat interface and propagate with velocity $V_s = \sqrt{G/\rho}$. The damping coefficient C_t increases with increasing area of contact. At frequencies approaching zero (wavelengths tending to infinity), C_t is vanishingly small (Fig. 2), due to the destructive interference of the very long waves associated with antisymmetric (torsional) vibration (Gazetas 1983). On the other extreme, at high frequencies (small wavelengths), all the points of the interface act as independent sources, radiating one-dimensional waves that propagate perpendicular to the soil-basemat contact surface with velocity V_s; therefore, the contribution to radiation damping, dC_{tb} of the shear waves emanating from an elemental area dA_b at the soil-basemat interface can be expressed as

$$dC_{tb} \approx \rho V_s \, dA_b r_b^2 \quad \text{for } \omega \rightarrow \infty \qquad (3a)$$

where r_b = the distance of the element from the axis of rotation.

The asymptotic value of the damping coefficient can then be obtained by integration over the whole soil-basemat contact area A_b of the torsional moments produced by all these elemental forces, dC_{tb}, around the axis of rotation

$$C_{tb} \approx \rho V_s I_b \quad \text{for } \omega \rightarrow \infty \qquad (3b)$$

where I_b = second moment of area of the basemat about the axis of rotation z.

At intermediate frequencies, away from the foregoing two extremes, the radiation damping coefficient of a surface foundation could be generally expressed as

$$C_{t,\text{surf}} \equiv C_{tb} = \tilde{c}_b(\rho V_s I_b) \qquad (4)$$

where the dimensionless dashpot coefficient $\tilde{c}_b$ is found to be a function of excitation frequency and basemat shape. Using the numerical (BE) data, the following simple algebraic expression is developed for $\tilde{c}_b$:

$$\tilde{c}_b = 1 - \exp(-0.3 a_o^{1.7}\, \Psi^{0.6}) \qquad (5a)$$

where

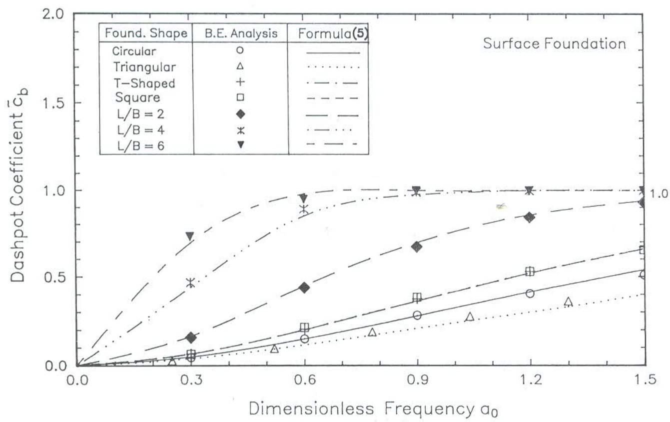


FIG. 2. Dashpot Coefficient, $\tilde{c}_b$, of Surface Foundations

$$a_o = \frac{\omega B}{V_s} \qquad (5b)$$

and

$$\Psi = \frac{I_b}{B^4} \text{ for circular foundations} \qquad (5c)$$

$$\Psi = \frac{I_{BL}}{B^4} \text{ for any other foundation shape} \qquad (5d)$$

where $2B$ and $2L$ = the width and length of the circumscribed rectangle (see Fig. 1); I_b = the second moment of area of the actual basemat surface; and I_{BL} = the second moment of area of the circumscribed rectangle, about the axis of rotation z

$$I_{BL} = \frac{4}{3} BL(B^2 + L^2) \qquad (5e)$$

In Fig. 2, radiation dashpot coefficients for circular, square, triangular, T-shaped, and rectangular foundations computed with the proposed formula [(5)] compare very satisfactorily against our numerical BE data. Notice that radiation damping is fairly sensitive to variations in both basemat shape and frequency. The dimensionless coefficient $\tilde{c}_b$ increases with increasing foundation aspect ratio L/B, as well as with increasing frequency; it asymptotically approaches unity as anticipated from (3b).

Finally, torsional radiation damping coefficients C_t of surface foundation predicted with developed formula are compared in Table 1 with both published results from the literature and our BE analysis. Please also note that results do not exist in the published literature for triangular, T-shaped, and rectangular foundations with $L/B > 4$; all numerical data in Table 1 and in Fig. 2 for these shapes are thus new.

Foundation in Open Trench

The effect on torsional radiation damping of placing a foundation without sidewalls at the bottom of a trench (Fig. 3) is studied for several basemat shapes using our BE formulation. The results for a square foundation, displayed in Fig. 4, show the variation of the trench factor, $F_{\text{tren}} = C_{t,\text{tren}}/C_{t,\text{surf}}$, as a function of frequency, for several depth ratios D/B and basemat shapes. This effect is significant only when the shear wavelength $\lambda_s = V_s/f$ (where f is the frequency in Hz) is large compared with the dimensions of the basemat. The value of frequency parameter a_o, at which the trench effect becomes practically negligible, depends on L/B and D/B ratios. This cutoff frequency becomes smaller with increasing values of L/B and D/B ratios.

The increase in torsional damping of a foundation basemat due to its placement at the bottom of a trench may be explained by considering the theory of wave interference. It is well established that because of destructive interference of waves (due to the phase differences between waves emitted from various points of the basemat-soil contact area) only a portion of the half-space in the form of a semi-infinite truncated cone is effective in transmitting the energy imparted to a basemat on an elastic half-space (Veletsos and Nair 1974; Dobry and Gazetas 1986). As a consequence, when the

TABLE 1. Damping Coefficient C_t: Comparison of Results for Surface Foundations

Basemat shape (1)	$a_o = \omega B/V_s$ (2)	$C_t(a_o)/GB^3$ — This Paper: Approximate formula [Eqs. (4) and (5)] (3)	This Paper: BE analysis (4)	Wong and Luco (1978) (5)	Wolf (1988) (6)	Day (1988) (7)
Circular	0.3	0.08	0.07	—	—	0.07
Circular	0.6	0.24	0.24	—	—	0.23
Circular	0.9	0.44	0.44	—	—	0.37
Circular	1.2	0.65	0.64	—	—	0.55
Circular	1.5	0.85	0.82	—	—	0.78
Square	0.3	0.18	0.17	0.15	0.16	—
Square	0.6	0.54	0.57	0.60	0.52	—
Square	0.9	0.97	1.02	1.00	1.00	—
Square	1.2	1.39	1.41	1.35	1.39	—
Square	1.5	1.76	1.74	1.61	1.73	—
Rectangular, $L/B = 2$	0.3	2.23	2.11	2.20	2.35	—
Rectangular, $L/B = 2$	0.6	5.98	5.85	5.80	6.17	—
Rectangular, $L/B = 2$	0.9	9.26	8.82	8.60	8.65	—
Rectangular, $L/B = 2$	1.2	11.40	10.86	10.30	10.60	—
Rectangular, $L/B = 2$	1.5	12.54	12.40	11.45	11.48	—
Rectangular, $L/B = 4$	0.3	39.85	40.67	38.60	43.34	—
Rectangular, $L/B = 4$	0.6	76.85	80.69	74.30	73.00	—
Rectangular, $L/B = 4$	0.9	88.53	90.23	88.60	86.80	—
Rectangular, $L/B = 4$	1.2	90.50	92.32	86.80	88.20	—
Rectangular, $L/B = 4$	1.5	90.65	91.86	84.60	89.57	—
Rectangular, $L/B = 6$	0.3	204.2	216.0	—	—	—
Rectangular, $L/B = 6$	0.6	290.1	281.2	—	—	—
Rectangular, $L/B = 6$	0.9	296.0	290.1	—	—	—
T-shaped	0.3	0.095	0.092	—	—	—
T-shaped	0.6	0.29	0.29	—	—	—
T-shaped	0.9	0.51	0.53	—	—	—
T-shaped	1.2	0.74	0.75	—	—	—
T-shaped	1.5	0.93	0.95	—	—	—
Triangular	0.5	0.094	0.097	—	—	—
Triangular	1.0	0.25	0.27	—	—	—

Note: G = shear modulus of soil.

wavelength is very small compared with the dimensions of the basemat, waves radiated from the basemat propagate only in the vertical direction perpendicular to the plane of the basemat. Thus, when the wavelength is small compared with the dimensions of footing basemat (or a_o is high) the presence of trench has no influence on the torsional radiation damping of the basemat. On the other hand, significant spreading of waves occurs when the wavelength is large compared with the dimensions of the basemat, and therefore the trench effect is significant.

To develop further understanding of how the trench effect arises, we consider the torsional shear stresses τ_{xy} induced in the soil during torsional vibration of a square foundation placed at the bottom of relatively deep trench, with $D = 2B$, at a dimensionless frequency $a_o = 1.5$. The imaginary (i.e., the out-of-phase with the excitation) part of τ_{xy}, which relates to radiation damping, has been computed with the BE formulation; Fig. 5 portrays its variation with distance x (measured from the center of the foundation), for three different depths: $z = 2B$ (i.e., along a line passing through the base), $z = B$ (at the middle of the trench), and $z = 2.5B$ ($0.5B$ below the base). Evidently, these components of shear stresses do not vanish

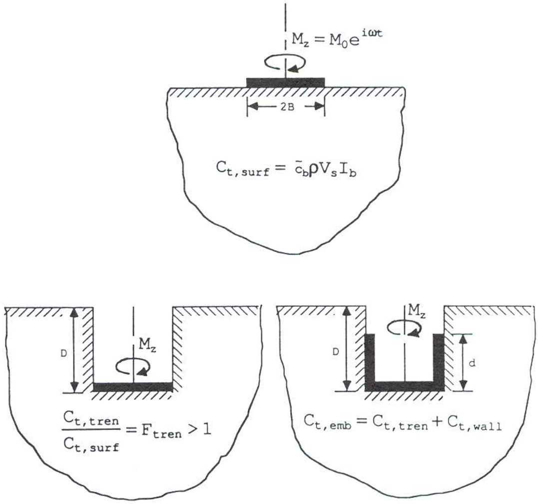


FIG. 3. Schematic Illustration of Effects of Embedment on Torsional Radiation Damping: (*a*) Surface Foundation; (*b*) Foundation in Open Trench; (c) Embedded Foundation with Full Sidewall-Soil Contact Over Height d

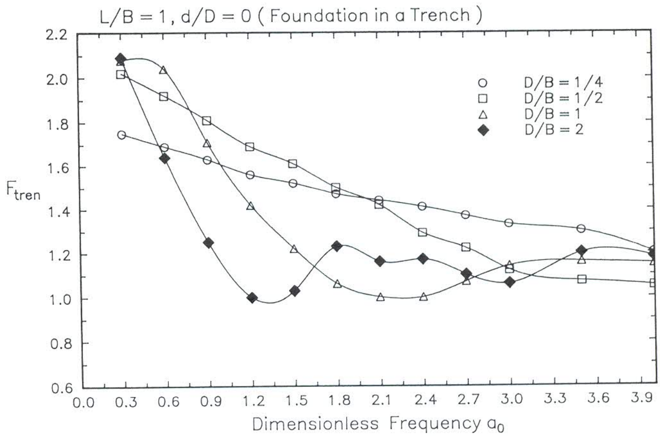


FIG. 4. Trench Factor F_{tren} **versus** a_o **for Square Foundation Placed in Open Trench**

above the basemat level, but they attain values comparable to those induced at and below the basemat level. This suggests that during torsional oscillation of a foundation placed in an open trench, in addition to radiation from the basemat, spreading of energy also takes place, with waves propagating in the soil above the basemat level.

Based on the complete set of results of the numerical investigation of the trench effect, the following simple expression has been developed, by trial and error, for estimating the damping of an arbitrarily shaped rigid foundation at the bottom of an open trench:

$$C_{t,tren} = F_{tren} C_{t,surf} \quad \text{(6a)}$$

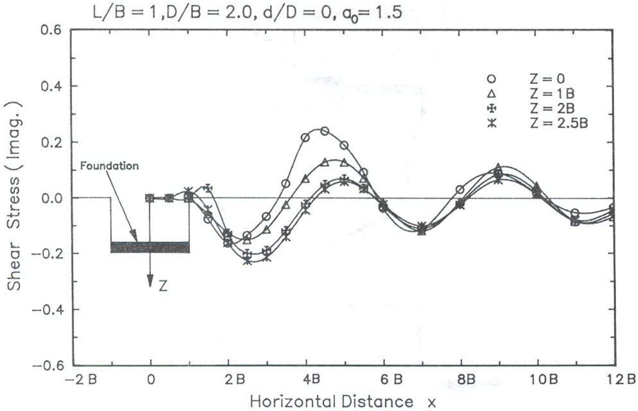


FIG. 5. Distribution of Shear Stress (Imaginary Part) in Soil during Torsional Oscillation of Square Foundation Placed in Open Trench

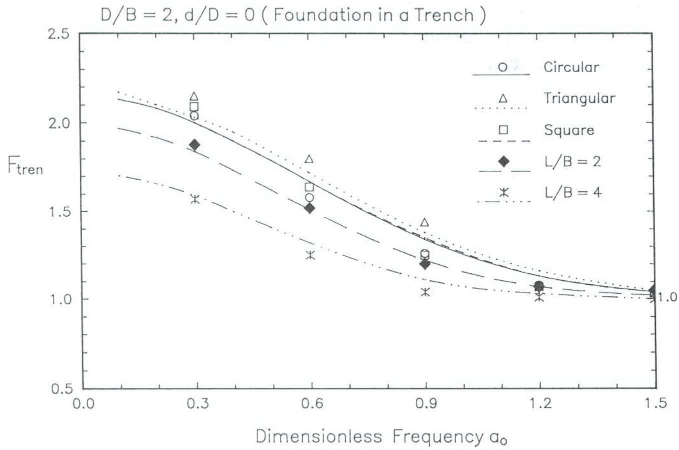


FIG. 6. Trench Factor F_{tren} for Various Basemat Shapes (D/B = 2, d/D = 0): Symbols Are for Numerical (BE) Results; Lines for Results of (6*a*)

where

$$F_{tren} = 1 + \frac{\left(\frac{D}{B}\right)^{0.2} \exp\left[-0.75\left(\frac{D}{B}\right)\left(\frac{L}{B}\right)^{0.3} a_o^2\right]}{\left[0.9 + 0.1\left(\frac{L}{B}\right)\right]^{1.4}} \quad \ldots\ldots\ldots\ldots\ldots (6b)$$

Fig. 6 compares the F_{tren} values for various basemat shapes obtained using the foregoing formula with values of the BE analysis, for an embedment ratio D/B = 2. Notice that the trench effect decreases with increasing frequency and aspect ratio (L/B) of the basemat.

Embedded Foundation—Radiation from Sidewall and Base

In the case of embedded foundations, in addition to shear waves originating at the basemat, shear and compression-extension waves are emitted from the vertical sidewall surfaces. A key simplification introduced in the proposed model is that the amount of damping contributed by the motion of each surface is independent of the presence and motion of the other surfaces of the foundation—an assumption for which indirect supporting evidence can be found in Chen et al. (1984). A similar assumption had been successfully employed for the other modes of vibration by Gazetas et al. (1985), Gazetas and Tassoulas (1987), and Fotopoulou et al. (1989).

Rectangular Foundation

The sidewalls of an embedded rectangular foundation in torsional oscillation perform two motions.

1. A rotation $\theta(t)$ around the vertical axis passing through the centroid of each particular sidewall-soil contact surface, equal to the angle of rotation of the (rigid) foundation. Owing to this motion, the wall emits compression-extension waves that are assumed to propagate with an apparent velocity $V_{ce} \approx V_{La} = 3.4V_s/[\pi(1 - \nu)]$, known as Lysmer's analog velocity (Gazetas and Dobry 1984; Gazetas and Tassoulas 1987). It is further assumed that the fact that the sidewalls are vertical rather than horizontal does not influence the high-frequency asymptotic value of radiation damping of the sidewalls, which is thus equal to the radiation dashpot of an identical rocking surface foundation, $\rho V_{La} I_{sw}$ (Fotopoulou et al. 1989). Therefore, the contribution to radiation damping by a particular sidewall surface due to its rotation can be expressed as

$$C_{w1} = \tilde{C}_1 \rho V_{La} I_{sw} \quad (7)$$

where the dimensionless coefficient $\tilde{c}_1$ = a function of a_o, d/D, D/B, and foundation shape; and I_{sw} = the second moment of area of the particular sidewall-soil contact surface about the vertical axis passing through its centroid.

2. A horizontal shearing displacement with amplitude equal to the product $\theta(t) \cdot \Delta$, in which Δ = the distance of the centroid of the particular sidewall surface from the foundation axis of rotation, z. In the case of a rectangular foundation, $\Delta = B$ and $\Delta = L$ for the long and short sidewall surfaces, respectively. As a result of this shearing motion, the sidewall emits torsional shear waves propagating with velocity V_s. Using arguments similar to those advanced in the preceding paragraph for the rotational mode, the dashpot force on a particular sidewall surface can be expressed as

$$F_d = \tilde{c}_2(\rho V_s A_{sw})\theta(t)\Delta \quad (8)$$

where $\tilde{c}_2$ = a function of a_o, d/D, D/B, and foundation shape; and A_{sw} = the area of the particular sidewall-soil contact surface. In the case of a rectangular foundation, $A_{sw} = 2Bd$ for each short surface, and $A_{sw} = 2Ld$ for each long surface.

This dashpot force, F_d, is assumed to act at the centroid of the particular sidewall-soil contact area. By considering the moment about the rotation axis z of the foundation, the contribution to radiation damping due to horizontal shearing along the sidewall-soil interface becomes

$$C_{w2} = \tilde{c}_2 \rho V_s A_{sw} \Delta^2 \qquad (9)$$

Finally, the total radiation damping of a rectangular embedded foundation can be obtained by summing up the contributions of all contact surfaces

$$C_{t,\text{emb}} = F_{\text{tren}} C_{t,\text{surf}} + C_{t,\text{wall}} \qquad (10)$$

where

$$C_{t,\text{wall}} = \Sigma(\tilde{c}_1 \rho V_{La} I_{sw} + \tilde{c}_2 \rho V_s A_{sw} \Delta^2)$$

$$C_{t,\text{wall}} = 4\rho d \left[\frac{1}{3} V_{La}(L^3 + B^3) + V_s BL(B + L) \right] \cdot \tilde{c}_w \qquad (11)$$

where the numerical BE data for all foundation types suggest: $\tilde{c}_1 \simeq \tilde{c}_2 \simeq \tilde{c}_w$. The approximate formula for the sidewall dashpot coefficient, $\tilde{c}_w$, is given later in (13).

Nonrectangular Foundations

For nonrectangular embedded foundations, the contribution of each (vertical) sidewall can be computed with the help of the circumscribed rectangular prism of dimensions $2B$ by $2L$ by d. A very simple relationship was found to approximately hold between the damping generated from the walls of this (fictitious) circumscribed rectangular prism and the damping produced by the sidewall surfaces of the actual foundation: Their ratio is roughly equal to the ratio of the polar moment of area of the circumscribed rectangular base to that of the actual basemat. A similar relationship had also been found to hold true for radiation damping in rocking (Fotopoulou et al. 1989). Thus, the total radiation damping of an arbitrarily shaped embedded foundation can be estimated by the following simple algebraic expression:

$$C_{t,\text{emb}} = F_{\text{tren}} \tilde{c}_b \rho V_s I_b + \tilde{c}_w (4\rho d) \left(\frac{I_b}{I_{BL}} \right) \left[\frac{V_{La}}{3} (L^3 + B^3) + V_s BL(B + L) \right] \qquad (12)$$

where I_{BL} = the polar moment of area of the circumscribed rectangle given in (5e); and I_b = the (true) polar moment of area of the actual basemat. The following simple expression for $\tilde{c}_w$ has been fitted by trial and error to the numerical BE data for various basemat shapes and embedment depths:

$$\tilde{c}_w = \left(\frac{D}{B} \right)^{0.1} \left(\frac{d}{D} \right)^{-0.35} \frac{a_o^{1.9}}{a_o^2 + \left(\dfrac{B^4}{I_b} \right)^{0.5}} \qquad (13)$$

A comparison between the predictions with the developed simple model for $\tilde{c}_w$ (13) and the numerical BE results is depicted in Figs. 7, 8, and 9, for rectangular embedded foundations with L/B = 1, 2, and 4, respectively. Evidently, the performance of the simple model is very good for all cases

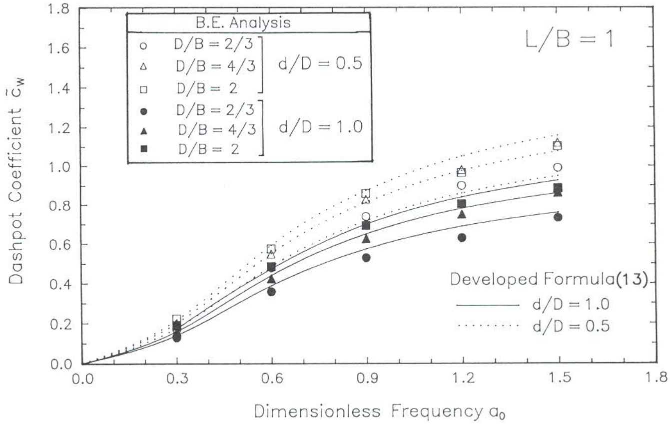


FIG. 7. Sidewall Dashpot Coefficient, $\tilde{c}_w$, of Square Foundations

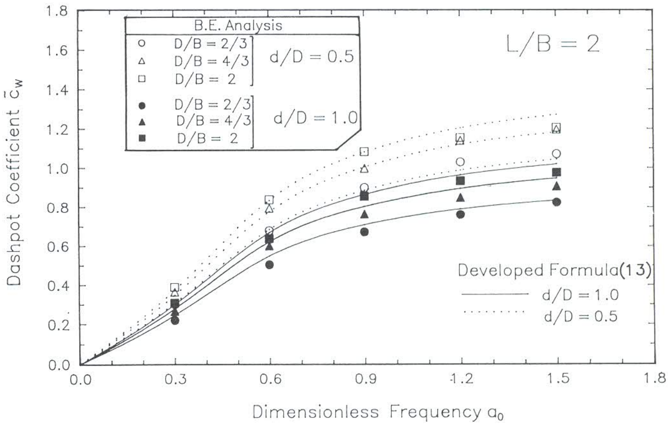


FIG. 8. Sidewall Dashpot Coefficient, $\tilde{c}_w$, of Rectangular Foundations with $L/B = 2$

presented in these figures. Moreover, this was also found to hold true for all the other cases studied (not shown herein).

The following important trend is worthy of note in these figures: The sidewall dashpot coefficient $\tilde{c}_w$ of an embedded foundation enjoying only partial sidewall-soil contact ($d/D < 1.0$) is always higher than that of a corresponding foundation having full sidewall-soil contact ($d/D = 1$)! This implies a greater efficiency in radiating wave energy from the deepest part of a sidewall. Thus, although the overall radiation damping, $C_{t,\mathrm{emb}}$, of a partially embedded foundation is always less than that of a corresponding fully embedded foundation [notice the factor d in (12)], the contribution of the sidewalls is not proportional to d; eliminating the upper part of a sidewall-soil contact (i.e., reducing d/D) would have a perhaps insignificant effect on damping. Hence, the engineer need not worry a great deal about separation or slippage that may occur near the ground surface [a similar

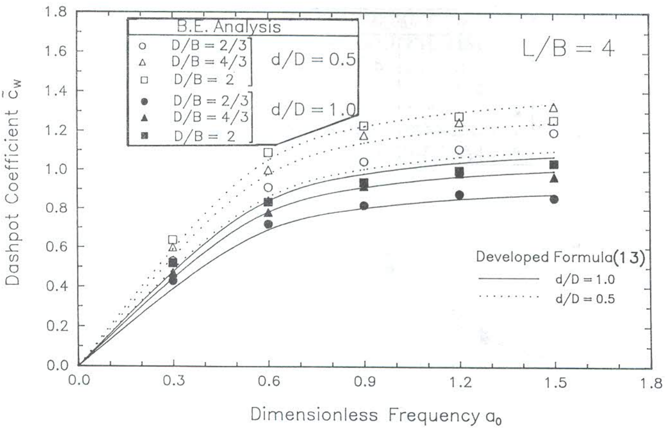


FIG. 9. Sidewall Dashpot Coefficients, $\tilde{c}_w$, of Rectangular Foundations with L/B = 4

TABLE 2. Damping Coefficient C_t: Comparison of Results for Fully Embedded Cylindrical Foundations

		$C_t(a_o)/GB^3$			
		This Paper			
Embedment ratio D/B (1)	$a_o = \omega B/V_s$ (2)	Approximate formula [Eqs. (12) and (13)] (3)	BE analysis (4)	Wolf (1988) (5)	Kausel and Ushijima (1976) (6)
0.5	0.3	0.53	0.51	0.48	0.43
0.5	0.6	1.53	1.42	1.39	1.62
0.5	0.9	2.45	2.33	2.19	2.24
0.5	1.2	3.12	2.97	2.86	2.74
0.5	1.5	3.58	3.39	3.28	3.19
1.0	0.3	0.98	0.97	0.94	1.0
1.0	0.6	2.80	2.86	2.63	3.14
1.0	0.9	4.38	4.40	4.36	4.44
1.0	1.2	5.46	5.53	5.72	5.25
1.0	1.5	6.16	6.05	6.34	5.93
2.0	0.3	1.94	2.12	2.01	2.23
2.0	0.6	5.50	5.86	5.68	6.03
2.0	0.9	8.52	8.64	8.45	9.13
2.0	1.2	10.57	10.29	10.24	10.55
2.0	1.5	11.92	11.39	11.65	11.32

Note: G = shear modulus of soil.

conclusion had been drawn by Fotopoulou et al. (1989) for damping of foundations rocking about their short axes].

Finally, torsional radiation damping of fully embedded cylindrical foundations predicted using the proposed formula [(12) and (13)] are compared, in Table 2, with published solutions of several researchers and our BE results. Again, the performance of the proposed model is very good.

Illustrative Example

The simple algebraic formulas proposed in this paper are used to obtain an estimate of the torsional radiation damping coefficient of the hypothetical embedded foundation sketched in Fig. 10. The chosen basemat is deliberately complicated to illustrate the capability of the method. The basemat is a truncated rectangle, placed 8 m below the ground surface. A 6-m sidewall, in perfect contact with the surrounding soil, is built around the periphery. The aspect ratio of the circumscribed rectangle is $L/B = 3$. Of interest is the value of C_t for an excitation frequency $\omega = 10$ rad/s.

The geometry and material parameters are given in Fig. 10, from which

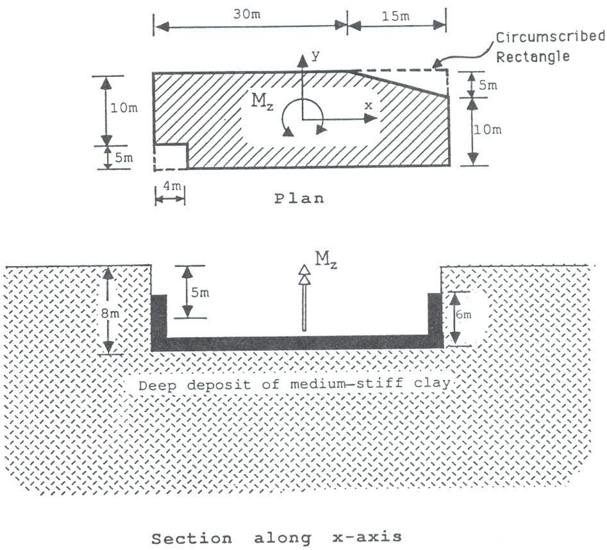
$$a_o = \frac{\omega B}{V_s} = \frac{10 \times 7.5}{194} = 0.39 \quad \text{(14)}$$

$$V_{La} = \frac{3.4 V_s}{\pi(1 - \nu)} = \frac{3.4 \times 194}{\pi(1 - 0.35)} = 323 \text{ m/s} \quad \text{(15)}$$

$$\frac{I_{BL}}{B^4} = \frac{126562.5}{(7.5)^4} = 40 \quad \text{(16)}$$

The radiation damping coefficient C_t is computed with the help of (12)

$$C_t = F_{\text{tren}} \tilde{c}_b \rho V_s I_b + \tilde{c}_w (4\rho d) \left(\frac{I_b}{I_{BL}}\right) \left[\frac{V_{La}}{3}(L^3 + B^3) + V_s BL(B + L)\right] \quad \text{(17a)}$$



Soil parameters	Foundation geometric parameters
$G = 68$ MPa	$L=22.5$m, $B=7.5$m, $L/B=3$
$\rho = 1.8$ Mg/m^3	$D/B=1.07$, $d/D=0.75$, $d/L=0.133$
$\nu = 0.35$	$A_b=617.5$m^2, $I_b=104308$m^4
$V_s = 194$ m/s	$I_{BL}=126562.5$m^4, $A_r=0.3644$
	$I_w/I_b=1.236$, $I_b/B^4=32.97$

FIG. 10. Illustrative Example: Geometry and Material Parameters

$$\tilde{c}_b = 1 - \exp\left[-0.3a_o^{1.7}\left(\frac{I_{BL}}{B^4}\right)^{0.6}\right]$$

$$= 1 - \exp[-0.3 \times (0.39)^{1.7} \times 40^{0.6}] = 0.42 \quad \ldots\ldots\ldots\ldots\ldots\ldots (17b)$$

$$F_{\text{tren}} = 1 + \frac{\left(\frac{D}{B}\right)^{0.2} \exp\left[-0.75\left(\frac{D}{B}\right)\left(\frac{L}{B}\right)^{0.3} a_o^2\right]}{\left[0.9 + 0.1\left(\frac{L}{B}\right)^{1.4}\right]}$$

$$= 1 + \frac{(1.07)^{0.2}\exp[-0.75 \times 1.07 \times 3^{0.3} \times (0.39)^2]}{[0.9 + 0.1 \times 3^{1.4}]} = 1.63 \quad \ldots\ldots (17c)$$

and

$$\tilde{c}_w = \left(\frac{D}{B}\right)^{0.1}\left(\frac{d}{D}\right)^{-0.35} \frac{a_o^{1.9}}{a_o^2 + \left(\frac{B^4}{I_b}\right)^{0.5}}$$

$$= (1.07)^{0.1}(0.75)^{-0.35} \frac{(0.39)^{1.9}}{(0.39)^2 + \left(\frac{1}{32.97}\right)^{0.5}} = 0.57 \quad \ldots\ldots\ldots\ldots\ldots (17d)$$

Therefore

$$C_t = 1.63 \times 0.42 \times 1.8 \times 194.0 \times (104308.0)$$

$$+ 0.57 \times 4.0 \times 1.8 \times 6 \times \left(\frac{104308.0}{126562.5}\right)$$

$$\times \left[\frac{323}{3} \times (22.5)^3 + (7.5)^3 + 194 \times 7.5 \times 22.5 \times (7.5 + 22.5)\right]$$

$$= 25 \times 10^6 + 5.56 \times 10^6 = 71 \times 10^6 \text{ KN}\cdot\text{m}\cdot\text{s} \quad \ldots\ldots\ldots\ldots\ldots\ldots (18)$$

Conclusion

This paper presents a comprehensive study on torsional radiation damping of embedded rigid foundations with a rigorous boundary element formulation, on the basis of which it develops closed-form simple algebraic formulas for estimating, inexpensively and reliably, the torsional radiation damping of rigid foundations embedded in a relatively deep homogeneous soil deposit. The developed model is versatile enough to handle foundations having any arbitrary (but solid) basemat shape and various degrees of contact between the vertical sidewalls and the surrounding soil: complete and perfect contact, partial symmetric and nonsymmetric contact, and no contact at all. For torsion of annular (ring-type) foundation basemat shapes, the reader is referred to Tassoulas (1981) and Veletsos and Tang (1988).

Appendix. References

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